Topological Hall effect in inhomogeneous superconductors

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We propose a possible mechanism of topological Hall effect in inhomogeneous superconducting states. In our scenario, the Berry phase effect associated with spatially modulated superconducting order parameter gives rise to a fictitious Lorentz force acting on quasiparticles. In the case of the Fulde-Ferrell-Larkin-Ovchinnikov state, the topological Hall effect is detected by applying an electromagnetic wave with a tuned wave number on a surface of the system.

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In a spin-singlet superconductor under an applied magnetic field, when the Pauli depairing effect dominates over the orbital depairing effect, spatially modulated superconducting order parameter is stabilized.[1,2] It has been discussed recently that this inhomogeneous superconducting state called the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state may be realized in a heavy-fermion system $CeCoIn_5$ and some quasilow-dimensional superconductors.[3–12] The possible realization of an analogous inhomogeneous superconducting state was also proposed for noncentrosymmetric superconductors, in which antisymmetric spin-orbit interaction combined with the Zeeman magnetic field stabilizes the helical vortex state.[13] It is an important issue to establish the realization of these exotic superconducting states in the above-mentioned systems experimentally. From this perspective, it is useful to study electromagnetic properties specific to the inhomogeneous superconducting states in detail, which may be utilized for the experimental identification of the modulated order parameter.[14,15] In this Rapid Communication, we demonstrate that a spatially varying superconducting order parameter characterizing the inhomogeneous state gives rise to distinct electromagnetic response caused by topological Berry phase effects.[16] In particular, under a certain circumstance, the topological Hall effect can be raised by a fictitious "Lorentz force" which is generated by the Berry phase effect associated with the inhomogeneous order parameter. It was discussed by Bruno *et al.*[17] that for electrons interacting with spin textures which possess a nonzero Berry curvature, the Hall effect is induced by the fictitious Lorentz force raised by the Berry phase effect. We here consider a possible analogous phenomenon in superconducting states with a spatially slowly varying order parameter. We note that the topological Hall effect considered in this Rapid Communication is a transport property of quasiparticles and we do not consider the Hall effect associated with supercurrents here.[18]

Our approach is based upon the quasiclassical method for the description of quasiparticle dynamics in superconducting states.[19–21] We extend the quasiclassical Eilenberger equation to take into account important Berry phase effects. The basic quantity with which we are concerned in the following argument is the single-particle Green's function for the superconducting state, from which dynamical properties can be derived

$$\hat{G}(x_1,x_2) = \begin{pmatrix} G(x_1,x_2) & -F(x_1,x_2) \\ F^\dagger(x_1,x_2) & \bar{G}(x_1,x_2) \end{pmatrix}, \qquad (1)$$

where $G(x_1,x_2)$ and $F(x_1,x_2)$ are, respectively, the normal and anomalous Green's functions, and $\bar{G}(x,x') = G(x',x)$, $x_1 = (\boldsymbol{r}_1, t_1)$, etc. The spatial modulation of the superconducting order parameter is expressed in terms of the center-of-mass coordinate $\boldsymbol{R} = \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}$. Fourier transforming the relative coordinate $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, we introduce the Wigner transformation of the Green's function

$$\hat{G}(\boldsymbol{k},\boldsymbol{R},\varepsilon_n) = \int d\boldsymbol{r} \int_0^\beta d\tau \hat{G}\left(\boldsymbol{R} + \frac{\boldsymbol{r}}{2}, \frac{\tau}{2}, \boldsymbol{R} - \frac{\boldsymbol{r}}{2}, -\frac{\tau}{2}\right) e^{i\varepsilon_n \tau} e^{i\boldsymbol{k}\boldsymbol{r}} \qquad (2)$$

with ε_n the fermionic Matsubara frequency. When there is a vector potential $\boldsymbol{A}_0$, the Gor'kov equation satisfied by $\hat{G}(\boldsymbol{k},\boldsymbol{R},\varepsilon_n)$ is

$$\left[\hat{\tau}_3 i\varepsilon_n - \varepsilon\left(\boldsymbol{k} - \frac{i}{2}\nabla_{\boldsymbol{R}} - e\boldsymbol{A}_0 \hat{\tau}_3\right) + \hat{\tau}_3 h - \hat{\Delta} - \hat{\Sigma}\right]\hat{G}(\boldsymbol{k},\boldsymbol{R},\varepsilon_n) = \hat{1}, \qquad (3)$$

where $\varepsilon(\boldsymbol{k})$ is the energy-band dispersion for electrons, $h = \mu_B H_z$ is the Zeeman magnetic field, $\hat{\tau}_\mu$ ($\mu = 1,2,3$) is the Pauli matrix in the particle-hole space, and $\hat{\Sigma}$ is the normal self-energy matrix which is diagonal in the particle-hole space, and includes effects of impurity scattering, and electron-electron interaction. We consider the spin-singlet pairing state with the gap function

$$\hat{\Delta}(x,x') = \begin{pmatrix} 0 & \Delta(\boldsymbol{R}) \\ -\Delta^*(\boldsymbol{R}) & 0 \end{pmatrix}. \qquad (4)$$

We expand the kinetic-energy term of Eq. (3) in terms of the spatial gradient $\nabla_{\boldsymbol{R}}$, and transform the basis of the particle-hole space as $\tilde{\hat{G}} = \hat{G}\hat{\tau}_3$. Then, Eq. (3) is rewritten into

$$\left[i\varepsilon_n + \hat{\tau}_3 \frac{i}{2}\boldsymbol{v}\nabla_{\boldsymbol{R}} + e\boldsymbol{v}\boldsymbol{A}_0 + h - \hat{\boldsymbol{\tau}}\cdot\hat{\boldsymbol{H}}_0 - \hat{\tau}_3 \hat{\Sigma}\right]\tilde{\hat{G}}(\boldsymbol{k},\boldsymbol{R},\varepsilon_n) = \hat{1} \qquad (5)$$

with $\hat{\boldsymbol{H}}_0 = [\Delta_1(\boldsymbol{R}), -\Delta_2(\boldsymbol{R}), \varepsilon(\boldsymbol{k})]$, $\Delta_1(\boldsymbol{R}) = \text{Re}\,\Delta(\boldsymbol{R})$, $\Delta_2(\boldsymbol{R}) = \text{Im}\,\Delta(\boldsymbol{R})$, and $\boldsymbol{v} = \frac{\partial\varepsilon(\boldsymbol{k})}{\partial\boldsymbol{k}}$. We diagonalize the fifth term $\hat{\boldsymbol{\tau}}\cdot\hat{\boldsymbol{H}}_0$

by applying the unitary transformation $\widetilde{\hat{G}}' = \hat{U}^\dagger(\boldsymbol{R})\widetilde{\hat{G}}\hat{U}(\boldsymbol{R})$; i.e., $\hat{U}^\dagger(\boldsymbol{R})\hat{\tau}\cdot\hat{\boldsymbol{H}}_0\hat{U}(\boldsymbol{R}) = E(\boldsymbol{k})\hat{\tau}_3$ and $E(\boldsymbol{k}) = \sqrt{\varepsilon^2(\boldsymbol{k}) + |\Delta(\boldsymbol{R})|^2}$

$$\left[i\varepsilon_n + \frac{i}{2}\hat{U}^\dagger\hat{\tau}_3\hat{U}\boldsymbol{v}\nabla_{\boldsymbol{R}} + \frac{\boldsymbol{v}}{2}\hat{A}_{\mathrm{f}} + e\boldsymbol{v}A_0 + h - E(\boldsymbol{k})\hat{\tau}_3 \right.$$
$$\left. - \hat{U}^\dagger\hat{\tau}_3\hat{\Sigma}\hat{U} \right]\widetilde{\hat{G}}' = \hat{1}. \tag{6}$$

Here, the unitary transformation applied to the spatial gradient term of Eq. (5), $\hat{\tau}_3\frac{i}{2}\boldsymbol{v}\nabla_{\boldsymbol{R}}$, gives rise to a fictitious vector potential $\hat{A}_{\mathrm{f}} = i\hat{U}^\dagger\hat{\tau}_3\nabla_{\boldsymbol{R}}\hat{U}$, which is a 2×2 matrix in the particle-hole space. At this stage, we apply the adiabatic approximation, assuming that the transition between the electron band with the energy $E(\boldsymbol{k})$ and the hole band with the energy $-E(\boldsymbol{k})$ is suppressed, and neglect the off-diagonal terms of $\hat{U}^\dagger\hat{\tau}_3\hat{U}$ and $\hat{A}_{\mathrm{f}}$; i.e.,

$$\hat{U}^\dagger\hat{\tau}_3\hat{U} \to \frac{\varepsilon(\boldsymbol{k})}{E(\boldsymbol{k})}\hat{\tau}_3, \tag{7}$$

$$\hat{A}_{\mathrm{f}} \to \frac{\varepsilon(\boldsymbol{k})}{E(\boldsymbol{k})}(\boldsymbol{A}_{\mathrm{f}1} + \boldsymbol{A}_{\mathrm{f}2}\hat{\tau}_3) \tag{8}$$

with $\boldsymbol{A}_{\mathrm{f}1} = \frac{1}{2}\left[1 - \frac{E(\boldsymbol{k})}{\varepsilon(\boldsymbol{k})}\right]\nabla_{\boldsymbol{R}}\phi(\boldsymbol{R})$ and $\boldsymbol{A}_{\mathrm{f}2} = -\frac{i}{4}\frac{\nabla_{\boldsymbol{R}}|\Delta|^2}{E^2(\boldsymbol{k})}$. Here $\phi(\boldsymbol{R})$ is the phase of the gap function; $\Delta(\boldsymbol{R}) = |\Delta(\boldsymbol{R})|e^{i\phi(\boldsymbol{R})}$. The approximation in Eqs. (7) and (8) is most crucial in our argument for the realization of the Berry phase effect. The Berry phase effect appears when one restricts the Hilbert space within a subspace in which the change in the phase of the wave function is regarded as an adiabatic one. Here, we restrict the Hilbert space within the electron band or the hole band. Then, suppressing the transition between the electron band and the hole band, one can neglect the off-diagonal elements of Eq. (6), as done in Eqs. (7) and (8). This approximation is valid at sufficiently low temperatures because of the energy difference between the electron band and the hole band due to the Zeeman splitting. We will discuss the validity of the approximation in more detail in the last part of this Rapid Communication. As a result, the diagonal component Eq. (8) can be regarded as fictitious U(1) gauge fields acting on quasiparticles. Within this approximation, $\widetilde{\hat{G}}'$ is diagonal; $\widetilde{\hat{G}}' = \mathrm{diag}(\widetilde{G}'_+, \widetilde{G}'_-)$. Each component satisfies

$$\left[i\varepsilon_n - \sigma E\left(\boldsymbol{k} - \frac{i}{2}\nabla_{\boldsymbol{R}} - \sigma\frac{\boldsymbol{A}_{\mathrm{f}1}}{2} - \frac{\boldsymbol{A}_{\mathrm{f}2}}{2} - \sigma e\frac{E(\boldsymbol{k})}{\varepsilon(\boldsymbol{k})}A_0\right) + h \right.$$
$$\left. - \widetilde{\Sigma}'_\sigma \right]\widetilde{G}'_\sigma = 1 \tag{9}$$

with $\sigma = \pm$, and $\widetilde{\Sigma}'_\sigma$ is the diagonal component of $\hat{U}^\dagger\hat{\tau}_3\hat{\Sigma}\hat{U}$. In Eq. (9), we have rewritten the derivative term and the vector potential terms into a gauge-invariant form.

To simplify the analysis, we assume that A_0 is a time-dependent uniform field, which yields an electric field. It is straightforward to generalize the following analysis to the case that A_0 also produces a magnetic field. To solve Eq. (9) for $\widetilde{G}'_\sigma$, we follow the quasiclassical approach developed by

Eilenberger. We extract the left-hand Gor'kov equation from the right hand side of Eq. (9), expand it in terms of the spatial gradient $\nabla_{\boldsymbol{R}}$ up to the second order, and integrate each term over the energy dispersion $\varepsilon_k \equiv \varepsilon(\boldsymbol{k})$. From the second term of Eq. (9), we obtain

$$\int \frac{d\varepsilon_k}{\pi}\left[i\sigma\frac{\varepsilon_k}{E(k)}\boldsymbol{v}\widetilde{\nabla}_{\boldsymbol{R}}\widetilde{G}'_\sigma + i(\boldsymbol{v}\times\boldsymbol{B}_{\mathrm{f}})\frac{\partial\widetilde{G}'_\sigma}{\partial\boldsymbol{k}_\parallel} \right], \tag{10}$$

where $\boldsymbol{B}_{\mathrm{f}}$ is the Berry curvature $\boldsymbol{B}_{\mathrm{f}} = \nabla\times\boldsymbol{A}_{\mathrm{f}1}$, the explicit expression of which is

$$(\boldsymbol{B}_{\mathrm{f}})_\alpha = -\epsilon_{\alpha\beta\gamma}\frac{1}{4E^2(k)}\frac{\partial|\Delta|^2}{\partial R_\beta}\left(\frac{\partial\phi}{\partial R_\gamma} - 2eA_{0\gamma}\right) \tag{11}$$

and $\widetilde{\nabla}_{\boldsymbol{R}}$ is the derivative with respect to $\boldsymbol{R}$ under the constraint that $E(k - \sigma\frac{A_{\mathrm{f}1}}{2} - \frac{A_{\mathrm{f}2}}{2} - \sigma e\frac{E_k}{\varepsilon_k}A_0)$ is fixed. $\boldsymbol{k}_\parallel$ is the momentum parallel to the Fermi surface. Note that the $\boldsymbol{A}_{\mathrm{f}2}$ term of Eq. (9) is a pure gauge, and does not give a nonzero Berry curvature. The second term of Eq. (10) is the fictitious Lorentz force term, the origin of which is the topological Berry phase effect raised by the spatial modulation of the superconducting order parameter. From Eq. (11), we see that the fictitious magnetic field is nonzero only when both the amplitude and the phase of the superconducting gap are spatially modulated. Thus, the topological Hall effect does not occur for the Fulde-Ferrel state and the helical vortex phase, in which only the phase of the superconducting gap is modulated.[2,13] It is also noted that the fictitious magnetic field Eq. (11) has a gauge-invariant form.

In the standard quasiclassical approach, the Gor'kov equation is recast into the Eilenberger equation for the normalized Green's function

$$\widetilde{g}'_\sigma = \int \frac{d\varepsilon_k}{\pi}\widetilde{G}'_\sigma. \tag{12}$$

However, unfortunately, the first term of Eq. (10) cannot be expressed in terms of the normalized Green's function because of a strongly varying factor $\varepsilon_k/E(k)$, which stems from the use of the transformed Green's function $\widetilde{G}'$ instead of the standard Green's function $\hat{G}$. To avoid this difficulty, we restrict our argument within the case with a uniform current and discard this term. For the second term of Eq. (10), we evaluate the integral over ε_k in the following manner:

$$\int \frac{d\varepsilon_k}{\pi}(\boldsymbol{v}\times\boldsymbol{B}_{\mathrm{f}})\frac{\partial\widetilde{G}'_\sigma}{\partial\boldsymbol{k}_\parallel} \approx (\boldsymbol{v}\times\widetilde{\boldsymbol{B}}_{\mathrm{f}})\frac{\partial\widetilde{g}'_\sigma}{\partial\boldsymbol{k}_\parallel} \tag{13}$$

with

$$\widetilde{\boldsymbol{B}}_{\mathrm{f}} = \begin{cases} \boldsymbol{B}_{\mathrm{f}}|_{\varepsilon_k=0} & \text{for } \Delta(\boldsymbol{R}) > h \\ 0 & \text{for } \Delta(\boldsymbol{R}) < h. \end{cases} \tag{14}$$

Then, the Eilenberger equation for the uniform current state satisfied by the normalized Green's function $\widetilde{g}'_\sigma(\varepsilon_n, \varepsilon_{n'})$ under the vector potential $A_0(t) = A_0(\omega_0)e^{i\omega_0 t}$ is given by

$$(i\varepsilon_n - i\varepsilon_{n'})\widetilde{g}'_\sigma(\varepsilon_n, \varepsilon_{n'}) + i(\boldsymbol{v} \times \widetilde{\boldsymbol{B}}_{\mathrm{f}}) \cdot \nabla_{\boldsymbol{k}_\parallel} \widetilde{g}'_\sigma(\varepsilon_n, \varepsilon_{n'}) + e\boldsymbol{v}A_0(\omega_0)$$

$$\times [\widetilde{g}'_\sigma(\varepsilon_n - \omega_0, \varepsilon_{n'}) - \widetilde{g}'_\sigma(\varepsilon_n, \varepsilon_{n'} + \omega_0)]$$

$$- \widetilde{\sigma}_\sigma(\varepsilon_n)\widetilde{g}'_\sigma(\varepsilon_n, \varepsilon_{n'}) + \widetilde{g}'_\sigma(\varepsilon_n, \varepsilon_{n'})\widetilde{\sigma}_\sigma(\varepsilon_{n'}) = 0. \quad (15)$$

Here $\widetilde{\sigma}_\sigma(\varepsilon_n)$ is the normalized self-energy $\widetilde{\sigma}_\sigma = \int \frac{d\varepsilon_k}{\pi} \widetilde{\Sigma}'_\sigma$. We have neglected effects of external fields on the self-energy. The second term of Eq. (15) is the fictitious Lorentz force term which gives rise to the topological Hall effect.

In the following, to be concrete, we consider the case of the FFLO state with the spatially modulated superconducting gap $\Delta(\boldsymbol{R}) = \Delta_0 \cos qx$, which is believed to be realized in CeCoIn$_5$ under an applied magnetic field parallel to the x axis.[4,5] We examine the Hall current parallel to the x axis induced by the topological Berry phase effect, when an electric field E_y is applied along the y axis. Note that since the Hall current is parallel to the external magnetic field in this situation, its is not difficult to distinguish between the topological Hall effect considered here and the ordinary Hall effect induced by the applied magnetic field in experimental measurements. As mentioned above, to obtain the nonzero $\boldsymbol{B}_{\mathrm{f}}$, we need the spatial modulation of the phase of the superconducting gap ϕ, as well as the amplitude modulation due to the FFLO state. To fulfill this requirement, we consider the situation that the electric field E_y induces a supercurrent parallel to the y axis, and thus $\partial_y \phi - 2eA_{0y} \neq 0$. We solve Eq. (15) for $\widetilde{g}'_\sigma$ in the vicinity of the superconducting transition temperature T_c. It is easily seen from Eq. (15) that $\widetilde{g}'_+$ and $\widetilde{g}'_-$ in the uniform current state are the same. Thus, we obtain the normalized Green's function in the original particle-hole space $\hat{g} = \mathrm{diag}(g, \bar{g}) = \widetilde{g}'_+ \hat{\tau}_3$. Then, the expression for the Hall current for $T \sim T_c$ is

$$J_x^{\mathrm{Hall}} = T \sum_n \int d\Omega_{\boldsymbol{k}} ev_x(g - \bar{g}) \Big|_{\omega \to 0}^{i\omega \to \omega + i\delta} \approx \frac{\sigma_n \tau \langle (\widetilde{\boldsymbol{B}}_{\mathrm{f}})_z \rangle}{m} E_y. \quad (16)$$

Here $\langle (\widetilde{\boldsymbol{B}}_{\mathrm{f}})_z \rangle$ is the spatial average of the fictitious magnetic field, σ_n is the normal-state conductivity, and τ is the relaxation time of electrons. Note that from the derivation described above, it is apparent that the topological Hall effect considered here is a nonlinear response to external fields. The bulk Hall current obtained above is nonzero only when the spatial average of the fictitious field $\widetilde{\boldsymbol{B}}_{\mathrm{f}}$ is nonzero. This implies that $\langle \partial_x |\Delta| \rangle = |\Delta(L_x)| - |\Delta(0)| \neq 0$; i.e., the magnitude of the gap function at the two opposite edges must be different. This condition crucially depends on extrinsic factors such as the geometry of a sample used for the measurement of the Hall effect, and pinning of the nodal plane of the FFLO state due to impurities. These extrinsic factors, unfortunately, makes it difficult to detect the Hall current experimentally. To avoid such extrinsic factors, one can use the scanning tunnel microscope (STM) measurement for the detection of the Hall effect. Even when the condition $\langle \partial_x |\Delta| \rangle = |\Delta(L_x)| - |\Delta(0)| \neq 0$ is not satisfied, the fictitious Lorentz force induced by the Berry curvature is balanced by the electrostatic force due to the topological Hall voltage which has a spatial dependence $V_{\mathrm{Hall}} \sim \cos qx$ in the above-mentioned

model. This electrostatic field gives rise to the inhomogeneous charge redistribution, which may be observed on the surface of the system via the STM measurement. It should be cautioned that the charge disproportion raised by the topological Hall effect cannot be described by Eq. (16) because it is assumed in its derivation that the current is spatially uniform.

However, more promising approach for the detection of the topological Hall effect is to exploit an electromagnetic wave $\boldsymbol{E}_0 e^{i(\omega t - kx)}$ applied on a surface of the system. For the setup considered above, the electromagnetic wave is a monochromatic plane-wave propagating along the x axis, and is linearly polarized so as that the electric field $\boldsymbol{E}_0$ is parallel to the y axis. We consider the situation that this electromagnetic wave is applied in addition to the static electric field parallel to the y axis, which is required to realize the nonzero fictitious field $\widetilde{\boldsymbol{B}}_{\mathrm{f}} \neq 0$. When the wave number k is chosen to be equal to q, the oscillating factor of the fictitious magnetic field $\widetilde{\boldsymbol{B}}_{\mathrm{f}}$ is cancelled out with that of the electromagnetic field, and hence, there is the net nonzero fictitious Lorentz force acting on quasiparticles. In this situation, we obtain the ac topological Hall current flowing along the x direction on the surface, which is easily detected. Since the induced Hall current is uniform in this case, the derivation of the expression of the Hall conductivity presented above is justified, and the Hall current is given by Eq. (16) with $\langle (\widetilde{\boldsymbol{B}}_{\mathrm{f}})_z \rangle$ replaced with $\langle (\widetilde{\boldsymbol{B}}_{\mathrm{f}})_z \cos qx \rangle$. It is noted that the above argument can be straightforwardly extended to the case with the orbital effect of magnetic fields.[10,11] We stress again that the direction of the topological Hall current considered here is parallel to the applied external magnetic field, which is required to realize the FFLO state. Thus, in experimental measurements, one can clearly discriminate between the topological Hall effect and the ordinary Hall effect of quasiparticles induced by the applied magnetic field.[22]

Finally, we discuss the validity of the adiabatic approximation which is crucial in our argument. The emergence of the Berry phase effect is due to the application of the adiabatic approximation; i.e., the transition between the electron band and the hole band is neglected, which is the central assumption in the derivation of the Gor'kov equation with the fictitious vector potential Eq. (9). This assumption is valid as far as there is an energy gap which separates the electron band and hole band, and temperature is sufficiently lower than the energy scale of the gap. However, in the FFLO state with the gap function $\Delta(x) = \Delta_0 \cos qx$, there are nodal planes of the superconducting gap at which $\Delta(x) = 0$. The existence of the nodal planes affects the energy spectrum of quasiparticles drastically. This issue was solved exactly in the case of the one-dimensional system,[15,23] and it was found that there is still an energy gap in the quasiparticle spectrum, which may validate the adiabatic approximation. However, in two and three dimensions with which we are concerned, it may be possible that the quasiparticle spectrum may become gapless because of the energy dispersion in the direction perpendicular to the x axis. Nevertheless, we can justify the adiabatic approximation applied to our system because of the following reason. Even if the superconducting gap vanishes at the nodal plane of the FFLO state, there is still an energy

gap between the electron band with up (down) spin and the hole band with down (up) spin in the vicinity of the Fermi level because of the Zeeman splitting. Thus, for temperatures much lower than the Zeeman energy scale, the transition between these two bands is suppressed, and hence, the adiabatic approximation is properly applied. Although we derived the expression for the topological Hall conductivity Eq. (16) only in the vicinity of the transition temperatures, the Hall effect is more clearly observed at sufficiently low temperatures.

In summary, we have demonstrated that the topological Hall effect of quasiparticles can be raised by the Berry phase effect associated with the spatially slowly varying superconducting order parameter which is realized in the FFLO state. The experimental detection of this effect which is feasible with the use of an ac electromagnetic field applied on a surface of the system may provide an evidence of the realization of the inhomogeneous superconducting state.

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